

Exercise 35

Evaluate the integral.

$$\int_1^2 \frac{v^3 + 3v^6}{v^4} dv$$

Solution

Rewrite the integrand by splitting up the fraction and writing it in a form that can be integrated easily.

$$\begin{aligned}\int_1^2 \frac{v^3 + 3v^6}{v^4} dv &= \int_1^2 \left(\frac{v^3}{v^4} + \frac{3v^6}{v^4} \right) dv \\ &= \int_1^2 \left(\frac{1}{v} + 3v^2 \right) dv\end{aligned}$$

The term on the left is the derivative of the natural logarithm (page 218), and the term on the right is the derivative of v^3 .

$$\begin{aligned}\int_1^2 \frac{v^3 + 3v^6}{v^4} dv &= \int_1^2 \left[\frac{d}{dv}(\ln v) + \frac{d}{dv}(v^3) \right] dv \\ &= \int_1^2 \frac{d}{dv}(\ln v) dv + \int_1^2 \frac{d}{dv}(v^3) dv\end{aligned}$$

Apply the second part of the fundamental theorem of calculus.

$$\begin{aligned}\int_1^2 \frac{v^3 + 3v^6}{v^4} dv &= (\ln v) \Big|_1^2 + (v^3) \Big|_1^2 \\ &= (\ln 2 - \ln 1) + (2^3 - 1^3) \\ &= 7 + \ln 2\end{aligned}$$